Physics 200 Name KEY 72 pts

Lab Time (circle one): 9:00am 12:00pm 3:00pm

Exam 1 – Force and Momentum

September 27, 2012

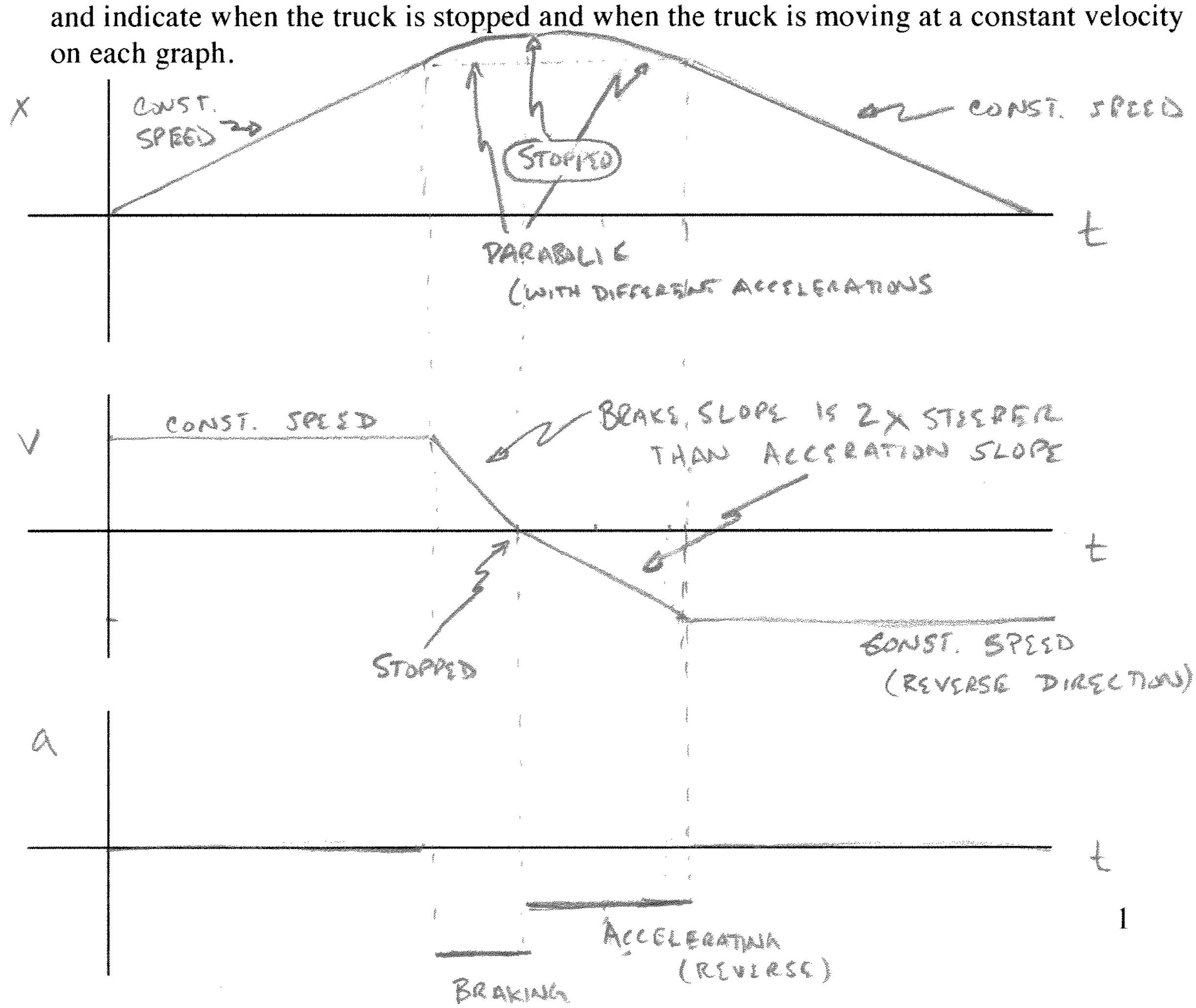
This is a closed book examination. You may use a both sides of a 3"x5" notecard (or one side of a 4"x6" notecard) with equations, concepts or other soothing sonnets. There is extra scratch paper available. Your explanation is worth ¾ of the points. Explain your answers!

A general reminder about problem solving:

1. Draw a picture then create a simplified free body diagram with all forces
2. Write down what you know including coordinate frame
3. Write down what you don't know and/or want to know
4. List mathematical relationships
5. Simplify and solve
6. Check your answer – Is it reasonable? Are units correct?

- Show all work!

1. [12 PTS] You are driving your truck down the street at exactly the speed limit on the way to your physics exam. Approaching a stop sign you applying a constant braking force. As soon as the truck is stopped you put the truck in reverse (since you realize you forgot your physics notecard) and back up at a constant acceleration till you reach the speed limit. The magnitude of your braking acceleration is twice your truck's reverse acceleration. Neglecting friction, draw $x(t)$, $v_x(t)$ and $a_x(t)$ for your truck. Completely label your graphs and indicate when the truck is stopped and when the truck is moving at a constant velocity on each graph.



2. [4 PTS] Recently the international space station (ISS) was moved into a higher altitude orbit around the earth (roughly 400 km). Many objects appear to float inside the ISS – that is they are stationary with respect to the ISS. What is the gravitational acceleration of an object inside the ISS? Explain.
   a) Exactly 9.81 m/s^2
   b) Obviously zero (0 m/s^2)
   c) Greater than 9.81 m/s^2
   d) Less than 9.81 m/s^2
   e) Depends on the mass of the object

$F_g = G\frac{M_E m}{(R_E + r)^2}$ $r = 400\,km = 4\times10^5$

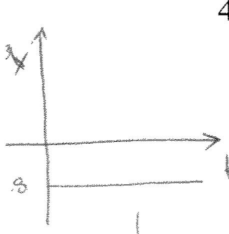
$R_E = 6.4\times10^6\,m$

$9.81\frac{m}{s^2} = g \equiv \frac{GM_E}{R_E^2}$ if $r \ll R_E$ then $g < 9.81$

3. [4 PTS] A deer runs out of the woods and hits the side of your car. The deer is much less massive than your car. It follows that the force the car exerts on the deer
   a) is greater in magnitude than the force the deer exerts on the car.
   b) is the same magnitude that the force the deer exerts on the car.
   c) is smaller in magnitude than the force the deer exerts on the car.
   d) can not be compared to the force the deer exerts on the car unless you know the angle of impact.

Forcesexerted by one object another object are equal in magnitude and opposite in direction

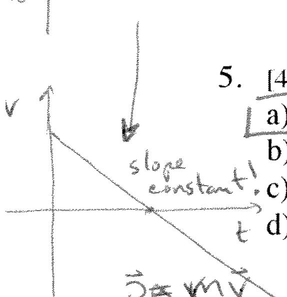
You throw a heavy small blue ball as hard as you can straight up into the air. Assume up is the positive direction and ignore air drag. The next three questions refer to this ball after it has left your hand. Please explain your answers (your explanation is worth ¾ of the points).

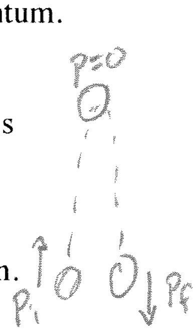


4. [4 PTS] The acceleration of the ball on the way up is
   a) 9.81 m/s^2 in the upward direction.
   b) zero (no acceleration).
   c) 9.81 m/s^2 in the downward direction.
   d) Can not tell. It depends on the initial momentum.

$F_g = mg$ $\downarrow g$

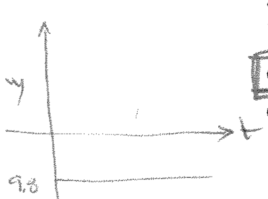
Gravitational acceleration is always constant and pointing toward the center of the Earth





5. [4 PTS] The momentum of the ball on the way up is
   a) positive (in the upward direction).
   b) zero.
   c) negative (in the downward direction).
   d) Can not tell. It depends on the initial position.

$\vec{p} = m\vec{v}$



6. [4 PTS] The acceleration of the ball at the very top of its throw (just before it starts back down) is
   a) 9.81 m/s^2 in the upward direction.
   b) zero (no acceleration).
   c) 9.81 m/s^2 in the downward direction.
   d) Can not tell. It depends on how high it was thrown.

see explanation for #4

7. [4 PTS] Two objects are dropped from a hot air balloon. The objects are identical except $m_B > m_A$. The first object ("A") reaches terminal velocity much faster than the second object ("B"). It follows that force due to air drag on object A is
   a) greater than the force due to air drag on object B
   b) the same as the force due to air drag on object B
   c) smaller than the force due to air drag on object B

$|F_D| = |F_g|$ at terminal velocity

$F_g = mg$

$F_{DA} = m_A g \quad m_A < m_B$

$F_{DB} = m_B g \quad F_{DA} < F_{DB}$

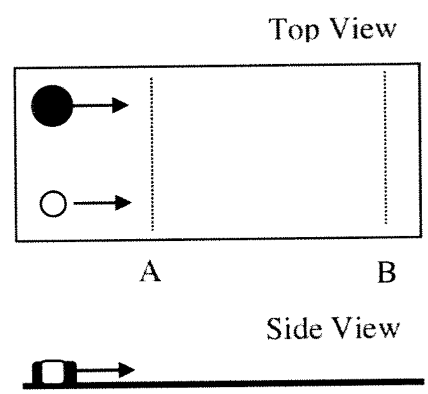
8. [4 PTS] A spring with a mass attached is hanging from a stand and oscillating with a period of 4 seconds. Which of the following would decrease the period to 2 seconds.
   a) Double the mass so $m_{new} = 2\ m_{old}$ $T \uparrow (\sqrt{2})$
   b) Attach an identical spring in parallel $T \downarrow (\sqrt{2})$
   c) Attach an identical spring in series $T \uparrow (\sqrt{2})$
   d) Decrease the mass so $m_{new} = \frac{1}{4}\ m_{old}$
   e) Double the mass so $m_{new} = 2\ m_{old}$ and attach an identical spring in parallel T same

$T = 2\pi\left(\frac{m}{k}\right)^{1/2}$

In parallel $k_{eff} = \sum_i k_i$

In series $\frac{1}{k_{eff}} = \sum_i \frac{1}{k_i}$

The next two questions refer to the diagram to the right and involve two disks. The disks, both initially at rest, are pushed with the same force for the same amount of time along a level surface. The push stops before either disk reaches line A. The black disk has twice the mass of the white disk. Assume that the surfaces are frictionless.



9. [4 PTS] Which disk has a greater change in momentum when it crosses line B?
   a) The white disk has a greater change in momentum.
   b) The black disk has a greater change in momentum.
   c) Both disks have the same change in momentum.
   d) Not enough information.

Same force and time $\rightarrow$ $F\Delta t = \Delta p$

momentum principle

10. [4 PTS] Which disk is traveling at a greater velocity when it crosses line B?
   a) The white disk is traveling at a greater velocity.
   b) The black disk is traveling at a greater velocity.
   c) Both disks have the same velocity.
   d) Not enough information.

$p = mv$

$p_B = m_B v_B = m_W v_W = p_W$

$m_B > m_W$

$v_B < v_W$

Please do the next two problems on additional paper or on problem solving sheets.

11. [12 PTS] At a highway intersection a blue car (mass = 2800 kg) collides with a silver pick-up truck (mass = 4700 kg). After the collision the car and truck slide along stuck together. The truck has a GPS unit that records its velocity. The truck's velocity just before the collision was $\langle -14, 0, 29 \rangle \frac{m}{s}$. After the collision the velocity of the stuck-together car and truck is $\langle 6.2, 0, 18.2 \rangle \frac{m}{s}$. The speed limit on both highways is 75 mph. Determine the car's velocity just before the collision. Was either the car or truck speeding?
(Note: 1 mile = 1609 meters)

12. [12 PTS] Suppose that you are navigating a spacecraft far from other objects. The mass of the spacecraft is $1.5\text{x}10^5$ kg (about 150 tons). The rocket engines are shut off and you are coasting along with a constant velocity of $\langle 0, 20, 0 \rangle \frac{km}{s}$. As you pass the location $\langle 1208, 1.5\times10^4, 2000 \rangle\ km$ you briefly fire side thruster rockets so that your spacecraft experiences a net force of $\langle 8.8\times10^4, 0, 0 \rangle\ N$ for 1 minute. You then continue coasting with the all rocket engines turned off. Assume the ejected gases have a mass that is small compared to the mass of the spacecraft. Determine your spacecraft's position and velocity 50 minutes after the side thruster rockets are finished firing.
(Note: 1 km = 1000 m)

Possibly useful mathematical relationships:

$$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$$
$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$$

Derivative of a polynomial $\frac{d}{du}Cu^n = nCu^{n-1}$ The Chain Rule $\frac{d}{dz}f(u) = \frac{d}{dz}u\frac{d}{du}f(u)$

Anti-derivative (integral) of a polynomial $\int Cu^n du = \frac{1}{n+1}Cu^{n+1} + const.$

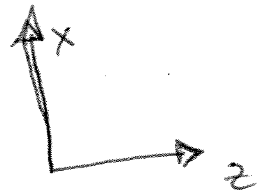
Useful Data:

Mass of Earth = $6\text{x}10^{24}$ kg
Radius of the Earth = $6.4\text{x}10^6$ m
G = $6.67\text{x}10^{-11}$ Nm^2/kg^2
Acceleration due to gravity at the surface of the earth is 9.81 m/s^2

Name: KEY
Lab Time:
Date: SEPT 27 2012
Test Code: EXAM 1
Problem #: 11

General Problem Solving Guide

List given information, define variables, sketch picture:



$v_c = ?$

$m_{CAR} = 2800\text{kg}$

$v = (6.2, 0, 18.2)$ m/s

$m_{TOT} = m_T + m_C$

$(-14, 0, 29)\ \text{m/s} = v_T$

$m_{truck} = 4700$

Simplify question, list target quantity:

Find velocity of car immediately before collision.

List all related quantitative relationships:

$\vec{P}_f = \vec{P}_i \qquad \Delta\vec{P} = 0 \qquad \text{since } \vec{F}_{NET} = \frac{\Delta\vec{P}}{\Delta t} = 0$

$\vec{p} = m\vec{v}$

Outline approach, sketch diagrams if needed (or sketch next to pictures above):

$\vec{P}_C + \vec{P}_T = \vec{P}_{TOT}$

$M_C\vec{V}_C + M_T\vec{V}_T = (M_C + M_T)\vec{V}$

solve for $\vec{V}_c$
use vector components

Obtain a general solution:

$$m_c\vec{v}_c + m_t\vec{v}_t = (m_c+m_t)\vec{v}$$

$$\vec{v}_t = (-14, 0, 29)\ m/s$$

$$v = (6.2, 0, 18.2)\ m/s$$

$$\vec{v}_c = \frac{(m_c+m_t)\vec{v} - m_t\vec{v}_t}{m_c}$$

$$= \vec{v} + \frac{m_t}{m_c}(\vec{v} - \vec{v}_t)$$

$$\begin{array}{r} (6.2\ ,\ 0\ ,\ 18.2)\ m/s \\ 1.68\ (\cancel{20.2}\ ,\ 0\ ,\ -10.8)\ m/s \\ \hline (40.1\ ,\ 0\ ,\ 0.06)\ m/s \end{array}$$

$$|v_t| = (\cancel{22}^{-14\,2} + 0^2 + \cancel{182}^{29\,2})^{1/2} = \cancel{282}^{32.2}\ m/s$$

$$= 32.2\,\frac{m}{s}\cdot\frac{1\,mile}{1609\,m}\cdot\frac{3600\,s}{1\,hr} = 72\ mph$$

$$|v_c| = (40.1^2 + 0^2 + 0.06^2)^{1/2} = 40.1\ m/s$$

$$= 40.1\,\frac{m}{s}\ \frac{3600}{1609}\ \frac{mph}{m/s} = 89.7\ mph$$

Check Units:

$$\frac{m}{s} = \frac{m}{s} + \frac{kg}{kg}\frac{m}{s} = m/s$$

✓

Check Limiting Cases:

$m_c \uparrow (\gg m_t)$ $\quad \vec{v}_c \approx \vec{v}$ ✓

$m_c \downarrow (\ll m_t)$ $\quad v_c \uparrow$ ✓

$v_t \approx v$ $\quad v_c \approx 0$

The lighter the car, the faster it must be going to change the truck's momentum.

Obtain a numeric solution:
(i.e. plug in the numbers)

$$v_c = (40.1, 0, 0.06)\ m/s$$

$|v_c| = 89.7\ mph \quad |v_T| = 72\ mph$

CAR is speeding!

Why is solution reasonable? Explain.

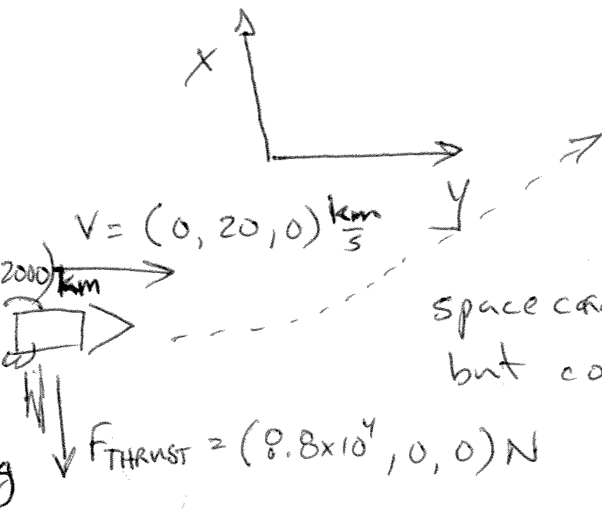
The units check out and limiting cases make sense – car has to be traveling fast to make collision move in a different direction from truck's momentum

Based on the Univ. of Minn. Cooperative Group Problem Solving Sheet, Modified By H. Cegla and S. Lindaas (2010) v2.2

Name:	KEY
Lab Time:	
Date:	SEPT 27 2012
Test Code:	EXAM 1
Problem #:	12

General Problem Solving Guide

List given information, define variables, sketch picture:



$V = (0, 20, 0) \frac{km}{s}$

$\vec{X} = (1208, 1.5 \times 10^4, 2000) km$

spacecraft pushed in the x direction but continues in the Y direction

$m = 1.5 \times 10^5 kg$

$F_{THRUST} = (8.8 \times 10^4, 0, 0) N$

$\Delta t_{THRUST} = 60 sec$

Simplify question, list target quantity:

Find position and velocity after 50 min.

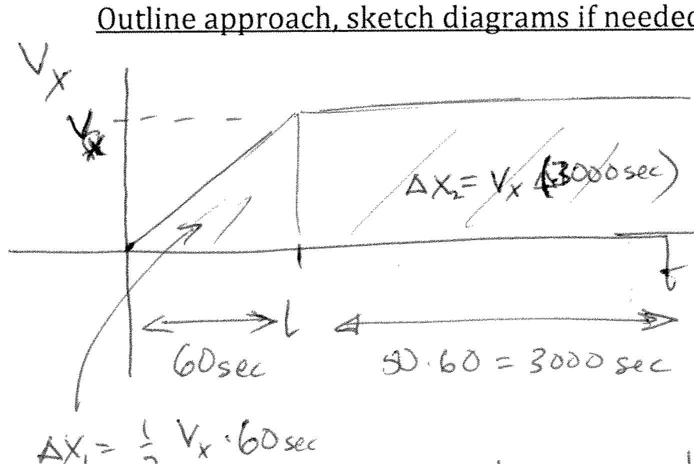
List all related quantitative relationships:

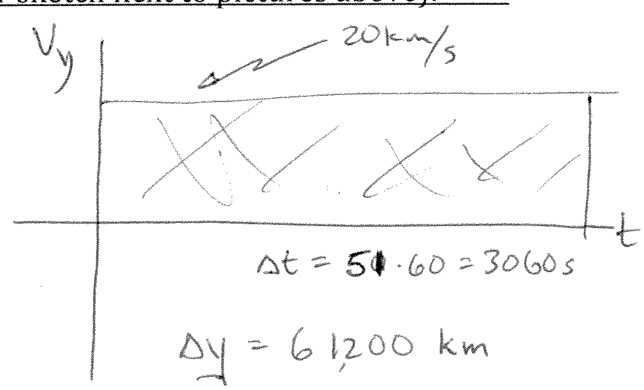
$\vec{F} = \frac{d\vec{p}}{dt}$ $\qquad \vec{V}_{AVG} = \frac{\Delta \vec{x}}{\Delta t}$

Use velocity vs time graphs to determine distance traveled $\Rightarrow$ area under curve

Add change in position to initial position

Outline approach, sketch diagrams if needed (or sketch next to pictures above):





Use vector components —

Obtain a general solution:

Thrust changes velocity in x-direction

$$F\Delta t = \Delta p = m v_x \qquad \left(\begin{array}{c} V = 0 \, \frac{km}{s} \\ \text{initially} \end{array}\right)$$

$$\frac{F\Delta t}{m} = v_x$$

$$\frac{(8.8\times10^4 \, N)(60 \, s)}{1.5\times10^5 \, kg} = 35.2 \, \frac{m}{s}$$

$$= 0.035 \, \frac{km}{s}$$

Not very fast compared to existing y-velocity

$$\Delta X_1 = 35.2 \, \frac{m}{s} \, (60 \, s) \cdot \frac{1}{2} = 1056 \, m$$

$$\Delta X_2 = 35.2 \, \frac{m}{s} \, (3000 \, s) = 105600 \, m$$

$$\Delta X = 106656 \, m = 107 \, km$$

$$\Delta y = 20 \, \frac{km}{s} \, (3060 \, s) = 61200 \, km$$

$$\vec{X}_0 = (1208, \; 1.5\times10^4, \; 2000) \, km$$

$$\Delta\vec{X} = (107, \; 6.12\times10^4, \; 2000) \, km$$

$$\vec{X}_f = (1315, \; 7.62\times10^4, \; 2000) \, km$$

Each component of vector adds separately!

Check Units:

$$N = \frac{kg \, \frac{m}{s}}{s} = kg \, \frac{m}{s^2} \quad ✓$$

$$m = \frac{m}{s} \cdot s = m \quad ✓$$

Check Limiting Cases:

$F \uparrow \quad v_x \uparrow$ ✓

$m \uparrow \quad v_x \downarrow$ ✓

$t \uparrow \quad v_x \uparrow$ ✓

Obtain a numeric solution:

(i.e. plug in the numbers)

$$\vec{V}_f = (0.035, \; 20, \; 0) \, km/s$$

$$\vec{X}_f = (1315, \; 7.62\times10^4, \; 2000) \, km$$

Why is solution reasonable? Explain.

The units check and the limiting cases make sense. Most of the change in position is in the y-direction since the y-velocity is much greater than x-velocity.